Dear Editor,

In an earlier letter [1] I outlined an analysis leading to a correspondence between Youden’s index (J) and the weight of evidence (log[LR(+)], the log-likelihood ratio of a positive test outcome) such that there is a monotonic increasing trend in log[LR(+)] from J = 0 towards J = 1. Here, for completeness, a graphical illustration of the correspondence is presented. The notation and terminology used here are the same as in [1]. The weight of evidence is a key component of evidence-based decision-making as Bayesian updating. We note here in passing that monotonic increasing trends in weight of evidence are also of interest in the context of decision-making at the level of neuronal responses to sensory stimuli [2].

Then to characterize the upper extent of the envelope, a large value of specificity is set and sensitivity allowed to vary, again calculating the corresponding log[LR(+)] and J values. The only real restriction here is to avoid indefinitely large log[LR(+)] (i.e., avoid specificity = 1). Here, the upper limit to specificity was taken to be 0.99, because values of both sensitivity and specificity are often presented to 2 d.p. in the literature. The resulting envelope is illustrated in Figure 1. In addition, Figure 1 is populated with data calculated from two previously-published studies that included the operational sensitivity and specificity values of a number of binary diagnostic tests [3, 4]. In each case, a monotonic increasing trend is illustrated.

We consider binary diagnostic tests. The specific scenario is one in which we know (only) the sensitivity and specificity at the operational threshold of a number of such tests. We calculate \( J = \text{sensitivity} + \text{specificity} - 1 \) and \( \log[LR(+)] = \log(\text{sensitivity}/(1 - \text{specificity})) \), all calculations here are carried out using natural logarithms) for each test. Now, our starting point is a graphical plot with log[LR(+)] on the ordinate and J on the abscissa. We can define an envelope within which most points representing corresponding (J, log[LR(+)]) pairs will lie, as follows. Recalling that both sensitivity and specificity are probabilities, the lower extent of the envelope is characterized by setting a large value of sensitivity and allowing specificity to vary, calculating the corresponding log[LR(+)] and J values.

Then to characterize the upper extent of the envelope, a large value of specificity is set and sensitivity allowed to vary, again calculating the corresponding log[LR(+)] and J values. The only real restriction here is to avoid indefinitely large log[LR(+)] (i.e., avoid specificity = 1). Here, the upper limit to specificity was taken to be 0.99, because values of both sensitivity and specificity are often presented to 2 d.p. in the literature. The resulting envelope is illustrated in Figure 1. In addition, Figure 1 is populated with data calculated from two previously-published studies that included the operational sensitivity and specificity values of a number of binary diagnostic tests [3, 4]. In each case, a monotonic increasing trend is illustrated.

Keywords
Youden’s index, log-likelihood ratio, diagnostic test

Summary
A correspondence between Youden’s index for rating diagnostic tests and the log-likelihood ratio of a positive test outcome is illustrated by data calculated from two previously-published studies of binary tests.

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Youden’s Index and the Weight of Evidence Revisited
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Figure 1  Correspondence between Youden’s index (J) and the weight of evidence (log[LR(+)], the log-likelihood ratio of a positive test outcome). The dashed lines show the envelope calculated as described in the text. The open circles represent 98 points calculated from data in [3] (for a further 10 points, specificity = 1, in which case log[LR(+)] is indefinitely large). The closed circles represent 84 points calculated from data in [4] (for a further two points, specificity = 1, and one further point was omitted after data-checks revealed an anomaly).
Note that the approach outlined above avoids use of $\log[LR(+)] = J$ as an approximation of the monotonic increasing trend in $\log[LR(+)]$ from $J = 0$ towards $J = 1$ [5]. I regard this as preferable for two reasons: 1) weights of evidence are not probabilities [6], so I think it is preferable to avoid equating $\log[LR(+)]$ and $J$, and 2) between the extremes there is not a one-to-one correspondence between $\log[LR(+)]$ and $J$ because diagnostic tests with equal values of $J$ will not necessarily have equal values of $\log[LR(+)]$ [1], so I think it is preferable to avoid writing the correspondence in the form of an equation. In addition, it is clear from Figure 4 that $\log[LR(+)] = J$ is not a good characterization of the correspondence illustrated there.

References

Erratum to: “Youden’s Index and the Likelihood Ratio Positive in Diagnostic Testing”

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Unfortunately, the letter “Youden’s Index and the Likelihood Ratio Positive in Diagnostic Testing” by D. Böhning contained an error. It contained the wrong Figure 4. The correct Figure is now supplied here.

D. Böhning sent us this statement: “I am grateful to Professor Hughes for pointing this out. There was no intention with my own letter to be critical on the letter by Professor Hughes or dispute its results but rather to add some thoughts of my own to the subject of interest.”

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Figure 4
Log-likelihood positive as a function of a cut-off value c for two normal distributions with equal unit variance and mean difference of 2