Assignment of Empirical Mode Decomposition Components and Its Application to Biomedical Signals

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Keywords
Empirical mode decomposition, correspondence problem, assignment problem, HRV, EEG

Summary
Objectives: Empirical mode decomposition (EMD) is a frequently used signal processing approach which adaptively decomposes a signal into a set of narrow-band components known as intrinsic mode functions (IMFs). For multi-trial, multivariate (multiple simultaneous recordings), and multi-subject analyses the number and signal properties of the IMFs can deviate from each other between trials, channels and subjects. A further processing of IMFs, e.g. a simple ensemble averaging, should determine which IMFs of one signal correspond to IMFs from another signal. When the signal properties have similar characteristics, the IMFs are assigned to each other. This problem is known as correspondence problem.

Methods: From the mathematical point of view, in some cases the correspondence problem can be transformed into an assignment problem which can be solved e.g. by the Kuhn-Munkres algorithm (KMA) by which a minimal cost matching can be found. We use the KMA for solving classic assignment problems, i.e. the pairwise correspondence between two sets of IMFs of equal cardinalities, and for pairwise correspondences between two sets of IMFs with different cardinalities representing an unbalanced assignment problem which is a special case of the k-cardinality assignment problem.

Results: A KMA-based approach to solve the correspondence problem was tested by using simulated, heart rate variability (HRV), and EEG data. The KMA-based results of HRV decomposition are compared with those obtained from a hierarchical cluster analysis (state-of-the-art). The major difference between the two approaches is that there is a more consistent assignment pattern using KMA. Integrating KMA into complex analysis concepts enables a comprehensive exploitation of the key advantages of the EMD. This can be demonstrated by non-linear analysis of HRV-related IMFs and by an EMD-based cross-frequency coupling analysis of the EEG data.

Conclusions: The successful application to HRV and EEG analysis demonstrates that our solutions can be used for automated EMD-based processing concepts for biomedical signals.

1. Introduction
Numerous methodological studies show that empirical mode decomposition (EMD) [1] approaches are particularly suitable to decompose a non-stationary multicomponent signal into its natural oscillatory components [2]. EMD decomposes a signal into a set of narrow-band components, which are known as intrinsic mode functions (IMFs), ‘by empirically identifying the physical time scales intrinsic to the data without assuming any basis functions’ [3]. IMFs are amplitude- and frequency-modulated signals, which are extracted by a sifting process. This sifting process can be seen as the result of a data-adapted filtering, i.e. the signal components are selected according to their natural frequency bands, meaning that no filter cut-off frequencies must be defined by the user. It can be demonstrated by simulations that the EMD acts as a dyadic filter bank, i.e. the center frequencies of the band passes have a dyadic distance [4]. Thus each IMF has specific frequency characteristics (spectrum, time-frequency representation), where IMF bands show overlaps. Because of these data-adaptive characteristics of EMD approaches, they have found wide application in the natural sciences, including biomedicine and neuroscience, as well as in the engineering sciences [5–8].

The adaptivity, i.e. the automatism of filtering which depends on the signal properties, is the first advantage of the EMD. A second advantage is that the non-linear properties of the natural signal components are pre-
served, which does not apply if linear filter techniques are used.

The basic algorithm of the EMD, introduced by Norden E. Huang [1] has been modified for bivariate [9] and multivariate versions (MEMD) [10] of signal decompositions as well as for bi-dimensional approaches in image analysis [11, 12]. Recent progress has addressed the so-called ‘mode-mixing’ problem, i.e. the alternative presence of several signal components of interest in the same IMF can be observed, which is frequently caused by the intermittency of segments with different signal properties, in particular different frequency characteristics. Advanced approaches such as ensemble empirical mode decomposition (EEMD) [13], complete ensemble empirical mode decomposition with adaptive noise (CEEMD) [14] and many other modifications reduce the degree of mode mixing.

For multi-trial, multivariate (multiple simultaneous recordings), and group (multiple subjects) analyses the correspondence problem (CP) occurs; in addition to differing spectral characteristics of IMFs the number and the properties of the IMFs between trials, channels and subjects can deviate from each other. Additionally, it is often observed that a continuous signal oscillation is split into two IMFs, i.e. its location in time alternates between both IMFs. Therefore two IMFs of the resulting IMF set can be characterized by similar frequency bands and thereby have qualitatively similar spectra. This is a specific mode mixing effect which aggravates the CP. Consequently, the resulting IMFs must be checked before further analysis (e.g. ensemble averaging) can follow. The CP is notable in biomedical signal analysis because measurement repetitions (trials) and ensemble averaging are effective means for the improvement of the signal-to-noise ratio or to reduce the influence of random signal events, respectively. Surprisingly, only few studies thus far have considered the CP. The semi-automated (SA) approach to resolve the CP, i.e. assignment decisions made by an expert (K.S.) which are based on analysis and comparison of the IMFs’ spectra [15], is extremely time-consuming and only reliable as well as practicable for small data sets (our application in EEG analyses with larger data sets). Consequently, an automation of this decision-making process is required. Up to now cluster analysis (CA) approaches have been used to assign trial or group-related IMFs which belong to each other [3, 10]. A natural requirement of assigning IMFs from different sources (e.g. subjects, trials) is that different IMFs of one source are assigned to pairwise different groups. In general, this cannot be guaranteed by using clustering procedures. If a multivariate approach is intended then MEMD can be used which reduces the CP to channel-related decompositions because MEMD mathematically enforces an iden-tical number of IMFs with regard to the recording channels. The channel-related IMFs have a direct correspondence to each other with regard to their signal characteristics [16]. Independent of the EMD version used, the CP must be solved in order to use IMFs with corresponding signal characteristics in further processing steps.

All these considerations provide the impetus to develop a fully automated approach for assigning IMFs from various sources, which would also overcome limitations of clustering methods. From the mathematical point of view, in some cases the CP can be transformed into an assignment problem which can be solved e.g. by the Kuhn-Munkres algorithm (KMA) [17] by which a minimal cost matching can be found. This approach has already been used to find optimal pairings of independent component analysis components [18]. Therefore, the main methodological goal of this study is to evaluate the efficiency of the KMA in solving the CP problem for group-related IMF analyses. For this purpose we use simulated data and the HRV data set because the assignment results of the SA approach can be used as a ‘gold standard’ for the comparison with the results derived from KMA as well as from the hierarchical CA approach [10]. The HRV data are derived from children with temporal lobe epilepsy; using one 5 min interval per child taken directly before the seizure onset. Additionally, we apply the KMA approach to a much larger EEG data set. Burst-interburst EEG signal patterns are used, which occur during quiet sleep in healthy newborns. Both data sets have already been used as bench-mark data for testing new algorithms [19]. By means of both applications, two main areas of biomedical signal analysis are considered: the cardiovascular-cardiorespiratory system and neurophysiological brain processes.

We have also integrated the KMA approach into complex analysis concepts to demonstrate that a comprehensive exploitation of the key advantages of the EMD after solving the CP is possible, i.e. the automated decomposition of a signal into its natural frequency components (signal-adapted filtering) and the preservation of non-linear signal characteristics.

The aim of the HRV analysis is to use EMD-related natural frequency components in a non-linear way, i.e. both key advantages of the EMD approach are exploited. HRV is composed of two main oscillatory signal components; the respiratory sinus arrhythmia (RSA) or high-frequency component and the low-frequency HRV component associated with Mayer waves in systemic blood pressure, where the task-force-related frequency ranges [20] are too unspecific to reveal changes in HRV characteristics before, during and after epileptic seizure [21]. Our own results [15, 21] have demonstrated that non-linear signal properties of signal components remain unaffected by EMD, thus a non-linear analysis of IMFs is appropriate.

The EEG data are investigated by an EMD-based cross-frequency coupling analyses (CFCA). Couplings between time-varying signal characteristics of different frequency bands are analyzed, including signal-to-envelope, envelope-to-envelope, envelope-to-frequency, and phase-to-phase couplings. In previous studies for filter-based CFCA we used bandwidths 1 Hz for neonatal sleep EEG [22] and 4 Hz for intensive care EEG [23]. The frequency-selective Hilbert transform (fsHT) can be applied for filter-based CFCA, which performs the filtering and the Hilbert transform (HT). The instantaneous amplitude (envelope), instantaneous frequency, and instantaneous phase can be computed from the analytic signal which can be constituted by using
The use of fixed filter bands is not optimal for many applications because if a filter band does not completely cover all variations in the instantaneous frequency of the corresponding EEG component, then the CFCA results become erroneous. However, the use of broader ‘standard’ frequency bands can result in a more serious problem, because a broadening carries the implicit danger that the instantaneous frequency and phase computation can be substantially impacted by artefacts [24]. Additionally, often predefined ‘standard’ frequency ranges, i.e. classical EEG frequency bands (delta, theta, alpha, etc.), are not suitable to characterize specific physiological or pathophysiological conditions, such as for the neonatal EEG [25, 26]. These are strong arguments for using the data-adaptive filter characteristics of the EMD in order to select natural signal components for further analysis. Using EMD for HT-based CFCA also has an additional advantage in that it was specially designed to serve as a preprocessing step for the HT; this combination is known as the Hilbert-Huang transform (HHT). This means that the fsHT is replaced by the HHT in our advanced CFCA concept.

As already mentioned above, CEEMD enables the separation of signal components with a simultaneous reduction of mode mixing (in comparison to EMD). Consequently, for this study CEEMD is used for decompositions with a low degree of mode mixing.

This article is organized as follows: In the section Materials simulated and HRV data are specified followed by a description of the methods in the section Methods. The concept of the EMD algorithm, which forms the basis for the CEEMD, is briefly explained. Thereafter, an overview of common CP types is given, where two CP types are relevant for our study and can be solved by using our approach. The derivation of the novel reference-based (using KMA) and the description of the comparative CA-based approach can be found at the end of this section. The results with respect to simulated and biomedical data, i.e. HRV and EEG data, are described in the section Results. Thereafter, the advantages and disadvantages of both approaches are discussed and conclusions with regard to further methodological developments are drawn.

2. Materials

2.1 Simulated Data

We generated five realizations of a univariate autoregressive process of order \( p = 0 \) described by

\[
x(m) = \sum_{r=1}^{p} a' \cdot x(m-r) + \varepsilon(m)
\]

with standard normally distributed variables \( \varepsilon(m) \) and autoregressive parameters as specified in Table 1, and \( m = 1, \ldots, 1000 \). The realizations are characterized by two pronounced spectral components at normalized frequencies 0.1 and 0.3 with varying amplitude ratios (Figure 1).

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2.2 Real Data

2.2.1 HRV Data

A group of 18 children each of whom had one epileptic seizure recording of at least 10 min (\( K = 18 \) seizures; median age: 9 years 4 months, range: 6 years 6 months to 18 years 0 months; median seizure length: 88 s, range: 52 to 177 s) was analyzed. Pre-surgical evaluation was performed at the Vienna pediatric epilepsy center following a standard protocol. The protocol was approved by the local Ethics Committee of the University Hospital Vienna. EEG was recorded referentially from gold disc electrodes placed according to the extended 10–20 system with additional temporal electrodes. One-channel ECG was recorded from an electrode placed under the left clavicle. EEG and ECG data were recorded referentially against CPZ, filtered (1 to 70 Hz), converted from analog to digital (sampling frequency 256 Hz, 12 bit), and stored digitally for further analysis. Video recordings of each seizure were reviewed to classify seizure type. Complex partial seizures were included, but not auras or generalized tonic-clonic seizures. Seizure onset and termination in the EEG were determined independently by two neurologists experienced in the field of epilepsy and clinical electrophysiology. EEG and ECG recordings including 10-minute epochs 5 minutes before (pre-ictal state) and 5 min after the seizure onset (seizure

![Figure 1](https://example.com/figure1.png)

**Figure 1** Selected amplitude spectra (#1, #3, #5) of the artificial data set
Assignment of EMD Components and post-ictal state) were stored for each seizure. QRS detection was performed after band pass filtering (10–50 Hz) and interpolation by cubic splines (interpolated sampling frequency 1024 Hz; according to (27)) to detect the time point of the maximum amplitude of each R-wave; the resulting series of events was used for the heart rate computation. The low-pass filtered event series (LPFES) was computed by applying the French-Holden algorithm [28].

The final HRV representation was obtained from the LPFES via multiplication with the sampling rate and with 60 beats per minute (bpm) and down sampled to 8 Hz. An artifact rejection was performed manually to minimize the influence of false QRS triggering. Finally, we used the pre-ictal state (300 s before seizure onset) for further analyses.

### 2.2.2 EEG Data

As recently described by Schiecke et al. [19], the EEG of a group of six full-term neonates (mean conceptual age 39.3 weeks, range 38–41 weeks; mean birth weight 3152 g, range 2670–3420 g; mean 5 min APGAR-score 9, range 8–10) was analyzed. Recordings were performed during sleep between 09.00 and 12.00 h, all neonates lay in an incubator at temperatures adapted to maintain normal body temperature and none showed any EEG abnormality. Eight-channel EEG (128 Hz sampling rate, international 10–20 system with electrodes Fp1, Fp2, C3, C4, T3, T4, O1, O2), heart rate, respiratory movements and EOG were recorded. Only the EEG recorded during quiet sleep was selected. The EEG was segmented by a trained physician; the burst onset was used as a fix point for a 10-s-interval. 4 s before (interburst) and 6 s after the burst-onset were considered. For the visual detection of the burst onset (amplitude criteria), the burst was defined as the simultaneous appearance of a group of high amplitude (> 50 µV), low frequency waves (0.5–3 Hz) in more than 75% of the recording channels. These waves are superimposed by low amplitude (< 50 µV), high frequency waves (4–15 Hz).

From our previous study (29) 10-s-intervals of the Fp1 recordings were analysed for each neonate; the burst-interburst-patterns starting with the beginning of the quiet sleep period were selected for analysis (17 burst-interburst patterns per neonate; minimal number of available 10-s intervals in one neonate; K = 102 for grand mean analysis of all neonates).

### 3. Methods

#### 3.1 Analysis Concepts

In Figure 2 the analysis concepts for both applications are depicted. In Figure 2A the approach for HRV analysis is represented. The spectrum (fast Fourier transform FFT) of each IMF is computed to provide signal characteristics for the assignment. The results of the SA approach are considered as ‘gold standard’, which are used as a reference for the results of the KMA and the CA approach. The assignment procedure provides assigned IMFs (IMF groups). The IMF groups of the KMA approach are used for further processing. Each of the resulting IMFs becomes embedded into the phase space from which the estimation of the point prediction error (PPE) for each IMF is derived. The PPE computation is described by Schwab et al. [30]. The PPE is a local estimation of the largest Lyapunov exponent, i.e. the estimation procedure yields a PPE time course. This time course is used for the time-variant characterization of the predictability (regularity, stability) of the IMF (signal), where sinusoidal signals are characterized by a PPE → 0 (deterministic signals correspond to a fully predictability) and
PPE-values for stochastic signals are very high (theoretically $\rightarrow \infty$).

In Figure 2B the analysis procedure for the EMD-based CFCA is shown. The EEG is decomposed using CEEMD. The resulting IMFs are assigned by the KMA on the basis of the IMFs’ amplitude spectra (FFT). Thereafter, for each IMF an analytic signal is computed, where the IMF itself is used as real component of its analytic signal. The corresponding imaginary component is computed by using the HT. The implementation of the HT and of the CFCA is described in detail by Witte et al. [22]. The IMF-related envelopes, instantaneous frequencies and phases can be calculated from the analytic signals [24]. CFCA is carried out between two IMFs (IMF groups), i.e. between two frequency ranges.

3.2 Empirical Mode Decomposition

EMD was first introduced in 1998 by N. E. Huang and co-workers [1] and subsequently named the Hilbert-Huang Transformation (HHT) since the resulting IMFs are processed by the HT. EMD decomposes a signal into a set of modes usually called intrinsic mode functions which satisfy two conditions: Firstly, the number of local extreme values and the number of zero crossings differ at most by one. This implies oscillating IMFs because a strict succession of local maximum, zero crossing, local minimum, zero crossing etc. is enforced. The second condition refers to envelopes of IMFs which ensures that the sum of upper and lower envelopes always equals zero. EMD is realized by an iterative procedure, which is continued until the residual function falls below a certain threshold. Thus, the number of resulting IMFs is not predefined and depends mainly on the decomposed signal. Meanwhile there have been many advancements and modifications of the classic EMD. All modifications have in common that they are time-variant signal-adaptive methods which can be used for the analysis of non-stationary and non-linear signals.

In 2009 Z. Wu and N. E. Huang introduced the ensemble EMD (EEMD) to reduce unwanted mode-mixing effects [13]. Here multiple realizations of white noise processes are added to the signal that is to be decomposed. The first IMF is obtained by averaging all first IMFs of the noisy realizations. After subtracting the common IMF from all noisy signals, the second common IMF is computed accordingly and so forth. This approach was extended by M. E. Torres and co-workers two years later by adding and also subtracting multiple noise processes [14]. This extension is known as the complete EEMD (CEEMD) and results in an improved spectral separation of modes and thereby a simplification of the correspondence problem, and reduced computational costs. We have applied CEEMD throughout the present study with a noise standard deviation of 0.05, 20 noise process realizations each, with a maximum of 100 sifting iterations [14].

3.3 The Correspondence Problem (CP)

Depending on the properties of the signals to be decomposed, the methodology used to extract the IMFs, and the specific analysis strategy, different forms of the correspondence problem arise. A schematic summary of the most important types related to group analyses of IMFs is shown in Figure 3. Let $x_1$ and $x_2$ be two univariate signals subjected to decomposition. In case (i), it is assumed that every IMF of $x_1$ has exactly one equivalent in the IMF set of $x_2$ and vice versa. Thus, the CP represents a classic assignment problem known as finding a maximum weight matching in a weighted bipartite graph. Case (ii) shows a case where decompositions of $x_1$ and $x_2$ result in different numbers of IMFs, a 1:n-assignment is still required allowing IMFs without assignment. Here, the number of correspondences is determined by the smaller set of IMFs. From the practical point of view, such constellations arise when one signal contains intrinsic oscillations which are absent in the other signal. In contrast to (ii), an 1:n-assignment is desired in case (iii). Herewith it can be enforced that every IMF of the larger set has exactly one equivalent in the smaller set of IMFs. Moreover, the natural requirement that every IMF of the smaller set has at least one equivalent in the larger set may be integrated. Thus, case (iii) is of particular interest when additional aspects of mode mixing are to be addressed. Case (iv) may be considered as a generalization of (ii) and considers that there might be IMFs without compatible equivalents in the other set of IMFs. In practice, in particular due to inter-individual differences, case (iv) may arise when signals of different subjects are processed. Finally, group analyses result in the multiple correspondence case (v),
where all previous pair-wise types are feasible.

The task of solving the correspondence problem is closely connected with a simultaneous processing of a multitude of IMFs. In fact, dependent upon particular signal properties, every signal is decomposed in IMFs of a greater or lesser extent. As a rule of thumb, one can expect that the CEEMD produces decompositions with usually at least ten IMFs for data containing noise. It implies that one can expect a set of at least 50 IMFs (precisely a total of 52 IMFs for five realizations) for our simulated data, a set of at least 180 IMFs (accurately a total of 212 IMFs for 18 subjects) for our HRV data, and at least 1020 IMFs (precisely a total of 1024 IMFs for six subjects with 17 trials each) for our EEG data. Obviously, even comparably small sample sizes of raw data lead to large IMF sets whose elements have to be assigned. Thus, solving the CP manually by an expert is impossible for relevant sample sizes and automated methods have to be used.

### 3.4 A Reference-based Approach as a Linear Assignment Problem

In this study, we focus on the solution of multiple CP incorporating the pair-wise types (i) and (ii). Let \( x_1, \ldots, x_N \), \( K \geq 2 \) be time series realizations, which are to be decomposed into sets of IMFs \( S_k = \{ I_{j,t,j} : j = 1, \ldots, J_k \} \) where \( I_{j,t,j} \) denotes the jth IMF of \( x_t \) and \( J_k \) is the number of IMFs resulting from the decomposition of \( x_t \). Let \( 1 \leq k \leq K \) be the index of a selected time series \( x_t \) serving later as a reference time series. Then, we aim to assign any set of IMFs \( S_k \) for all \( k \neq k' \) to the reference set \( S_r \). For it, we define the cost \( c_{ij} \) for assigning \( I_{j,t,j} \) to \( I_{i,t} \) by one minus the Pearson correlation coefficient between the amplitude spectra of \( I_{j,t,j} \) and \( I_{i,t} \). Using these settings, the correspondence problem is transformed into an assignment problem with rectangular matrices since \( S_r \) and \( S_r \) are not necessarily equal. It can be solved e.g. with an extension of the Kuhn–Munkres algorithm [31] that minimizes the costs \( m_{i,j} = \max(j_i, J_k) \) matching pairs. This means in terms of Figure 3 that the cases (i) and (ii) are encompassed.

Obviously, the overall assignment based on the entire sample \( \{ x_1, \ldots, x_N \} \) may depend on the choice of the reference signal \( x_t \) or set \( S_r \), respectively. From our experience, the best results are obtained when a reference set with maximum cardinality is selected. That is, it holds \( J_r \geq J_k \), which however does not necessarily specify a unique reference set, yet. In the case of multiple sets with maximum cardinality, the one with minimal total assignment cost is selected. In detail, if \( S_r \) is a possible candidate for a reference set, the total assignment costs are computed by \( M_k = \sum_{i,j} m_{i,j} \), and finally the inequality \( M_r \leq M_k \) should be fulfilled for all \( k \) with \( J_r = J_k \).

Speed and performance of the multiple correspondence procedure may also be improved because in the EMD approach the generated IMFs are roughly sorted according to decreasing frequency. Moreover, it can be observed in group analyses that the maximum number of IMFs \( \max(J_k) \) is only marginally larger than \( \min(J_r) \). Consequently, it is very unlikely or even impossible that \( I_{j,t} \) and \( I_{j',t} \) belong together, when \( |i - j| \) is large. Thus, for the assignment of \( S_r \) and \( S_k \) the cost modification \( c_{ij} = \infty \) if \( |i - j| > \max(J_r - J_k) + \delta \) is obvious, so we used \( \delta = 2 \) in our applications.

### 3.5 A Clustering-based Approach

In general, clustering-based methods have the advantage that they do not require the specification of a reference IMF set. On the other hand, it cannot be guaranteed that different IMFs of one set are mapped to pair-wise different clusters, which is ensured by the reference-based approach introduced above.

For the purpose of comparison we use an agglomerative hierarchical clustering approach with the weighted average distance as linkage criterion. To this end, in a first step all sets \( S_k \), \( k = 1, \ldots, K \) are pooled into one data set and the distance between every pair of objects in the pooled data set is computed. Again, one minus the Pearson correlation coefficient between the amplitude spectra of the IMFs was used. The number of clusters should obviously be set between \( \min(J_k) \) and \( \max(J_k) \).

Depending upon which data basis and EMD methodology used, the problem of assigning IMFs originating from one and the same signal into one cluster is more or less developed. From the practical point of view, these effects may be significantly reduced by a systematic modification of the distance matrix. This would provide higher or maximum distances (dissimilarities) for IMF pairs belonging to the same signal, respectively. Using the aforementioned metrics, the dissimilarities between IMFs to be assigned to different clusters are set to 2. The theoretical drawback of this modification is that the triangle inequality is no longer always fulfilled. However, we could not observe any negative impact on the clustering results.

### 4. Results

#### 4.1 Testing of the Assignment Performance of the KMA

##### 4.1.1 Simulated Data

To demonstrate the functionality of the reference-based assignment approach, we used \( K = 5 \) realizations of the univariate autoregressive process described in the Materials section. Here, it should be noted that the proposed methodology is not restricted to small data sets – the current \( K \) was chosen for the sake of clarity only. The signals were decomposed by CEEMD with a noise standard deviation of 0.05, 20 realizations each, and a maximum of 100 shifting iterations. An example of the first six IMFs \( I_1 = I_2 = 11, I_3 = I_4 = I_5 = 10 \) is shown in Figure 4. Here the two main oscillatory components around the frequencies 0.3 and 0.1 (Figure 1) are mainly contained in the first two IMFs each. To evaluate this approach, we established a ground truth assignment table, where beginning with \( I_1 \), an assignment could not be determined beyond a doubt (Table 2 left).

Using the reference-based approach, two sets of IMFs could be considered as reference set since \( J_1 = J_2 = 11 \). The overall assignment costs are \( M_1 = 9.54 \) and \( M_2 = 8.99 \), such that finally \( S_2 \) is used as a reference set. Two exemplary cost matrices are shown in Figure 5, whereby Figure 5A demonstrates a best case scenario, where the assignment problem can be easily solved manually. Figure 5B is a worst case example, where a subjective assign-
ment is no longer reliably solvable. The entire assignment table for the multiple correspondences is shown in the middle panel of ▶Table 2. Clearly excellent agreement with the ground truth is observable as far as comparisons are possible. A similar, however slightly impaired, result could be reached with the hierarchical clustering approach (▶Table 2 right). Moreover the aforementioned disadvantages of clustering-based approaches are clearly visible.

4.1.2 HRV Data

K = 18 signals were decomposed by CEEMD with a noise standard deviation of 0.05, 20 realizations each, and a maximum of 100 shifting iterations, where six decompositions resulted in 11 IMFs each, 10 signals were decomposed into 12 IMFs. The maximum number of 13 IMFs was achieved for two subjects (#1 and #10) yielding the final reference set $S_1$. As already seen in the simulated data set cost matrices revealed, that the low frequency IMFs are characterized by high spectral correlations resulting in low assignment costs. Additionally, a similar property may

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The problem of assigning multiple IMFs originating from one and the same signal is visible in group K (highlighted in dark gray). Moreover, using 10 clusters IMF 7 and IMF 8 of $x_1$ would be assigned to one cluster as well.
also be observed for the high frequency IMFs, which is caused by the CEEMD’s added noise. The impact of this effect depends on the frequency content of the signals. If high frequency components are missing the adding of noise introduces additional high frequency IMFs. In the case of the simulated data set, this effect is negligible because of the high intrinsic noise level of stochastic process realizations. In practice possible artificial high frequency IMFs can be easily identified, and are clearly separated from IMFs containing meaningful signal components.

Figure 6 shows the final assignment for multiple correspondences according to either the reference or cluster-based approach. The most prominent difference between the two approaches is that there is a more homogenous assignment pattern with the reference-based approach (Figure 6A). In particular, the most relevant IMFs 5–8 are very consistently mapped to their groups (E–H). In the case of the clustering approach, we could not avoid very sparse or full clusters, respectively (Figure 6B, e.g. cluster C, D, E, J, M). The fifth IMFs are consistently split into two clusters H and I (Figure 6B α), where IMFs 6 and 7 are combined into one cluster J (Figure 6B β). The problem of assigning multiple IMFs originating from one and the same subject is even more visible for the real data set in comparison to the simulated data (e.g. Figure 6B γ–ε).

### 4.1.3 EEG data

K = 102 signals were decomposed by CEEMD with a noise standard deviation of 0.05, 20 realizations each, and a maximum of 100 shifting iterations. Nineteen decompositions resulted in 9 IMFs each, 61 signals were decomposed into 10 IMFs, and 21 signals into 11 IMFs. The maximum number of 12 IMFs was achieved only for one signal, yielding the reference set $S_3$. As already seen in the simulated data set and in the HRV data the low frequency IMFs are characterized by high spectral correlations resulting in low assignment costs. Additionally, the high frequency IMFs are caused by the CEEMD’s added noise and 50 Hz noise.

Figure 7 shows the final assignment for multiple correspondences based on the reference-based approach. The most relevant IMFs 3–6 are very consistently mapped to their groups (C–F).

### 4.2 Further Analysis of the IMFs

#### 4.2.1 Point Prediction Error Analysis of the Assigned IMFs Derived from HRV

The average amplitude spectra of the reference-based approach are depicted in Figure 8A. The IMFs assigned to group F correspond to low-frequency rhythms known as Mayer-wave-associated HRV rhythms which occur with a center frequency of about 0.1 Hz. This component is an indicator of sympathetic and parasympathetic (vagal) influences on the HRV [20]. The IMFs assigned to group E correspond to the high-frequency component of the HRV which is dominated by the so-called respiratory sinus arrhythmia, which is mediated predominantly by vagal influences on the sinus node and therefore is often employed as an index of the HRV’s vagal control [32]. IMFs of group H are associated with very low frequency rhythms. The PPE courses of the HRV’s IMF groups is shown in Figure 8B. A slight trend to lower PPE values starting at about 200 s, i.e. 100 s before the seizure onset occurs (300 s), indicates a higher predictability (regularity, stability) of the assigned IMFs. This is consist-
ent with the findings of a previous study in which an interval which covered the subintervals before (preictal, used in this study), during and after the seizure was used [21]. As expected for this more stationary subinterval (in comparison to the ictal and postictal subintervals) the spectral characteristics of the four consecutive IMF groups E – H are more consistent with the definition of the ‘classical’ (task force recommendations [20]) HRV components. The most striking finding in this context is the fact that such a change of signal properties prior the seizure onset cannot be identified by using time-frequency and time-variant bispectral analysis.

4.2.2 EMD-based Cross-frequency Coupling Analysis of Neonatal EEG Patterns

In Figure 9 the results of the envelope-to-envelope CFCA for the EEG’s IMF groups C–F are represented which were computed automatically, i.e. without a defi-
The most striking result is that the peak-frequencies of the spectra of groups C–F are consistent with the mid-frequencies and bandwidths of the previously used frequency bands [22], where their definition was based on a priori knowledge derived from several studies with regard to frequency band analyses of the neonatal EEG, e.g. [33]. In Figure 9A the grand mean (GM) (K = 102 patterns; 17 burst-interburst patterns per neonate) spectra of group C–F are depicted in black (broad line) together with their standard deviations. The peak frequency of the GM spectrum of group C is located around 5 Hz and corresponds with the 4.1–5 Hz band in our previous study [22]. The GM peak frequency of group D is between 3 and 4 Hz and that of group E is at 2 Hz, which corresponds with the high-frequency bands 3.0–4 Hz and 1.6–2 Hz. The GM peak frequency of group F is at 1 Hz and the location of the peak corresponds with the previously used low-frequency band 0.5–1.5 Hz. The grand mean envelopes (± standard deviation) of groups C–F are depicted in Figure 9B. The envelope of group C show a sharp rise at the beginning of the burst (burst onset at 4 s), where the maximum is reached no later than about two seconds after the onset. The same can be obtained for the envelope courses of D–F, where their rise-onset is delayed relative to the rise of group C envelopes. The envelope-to-envelope correlations (not depicted) are fully consistent with those of our previous study [22] in which we used fixed, disjunctive, narrow frequency bands, i.e. the result is that a coupling between the low-frequency and the high-frequency components after a burst onset can be assumed. Envelope-to-frequency and frequency-to-frequency couplings are influenced by artifacts in the instantaneous frequency courses, i.e. the CFCA must be restricted to envelope-to-envelope couplings.

5. Discussion

Our study was initiated by the observation that for multi-trial, multivariate, and group EMD analyses the number and the properties of the IMFs between trials, channels and subjects can deviate from each other. Before a further processing of the IMFs can be performed, the correspondence of their signal properties must be checked, meaning that the correspondence problem must be solved. The successful application in HRV and EEG analysis demonstrates that our solutions can be used for automatic EMD-based processing concepts for bio-
medical signals. In a previous study we showed how complex a semi-automated IMF correspondence analysis for EEG data can be [16]. We used the results of spectral analysis of the MEMD-based IMFs to achieve an assignment by visual inspection. To the best of our knowledge, this is the first study which has considered the CP as an assignment problem which can be solved e.g. by the KMA. The few studies which have considered the CP thus far have used clustering algorithms. Therefore, we simultaneously applied clustering as a state-of-the-art approach in order to show the advantages and disadvantages of our approach.

The key advantages of the EMD (after solving the CP) are the automated decomposition of a signal into its natural frequency components (signal-adapted filtering) and the preservation of non-linear signal characteristics. Nevertheless, the possibility of other derivations of relevant frequency bands from the spectra has to be discussed; especially bandpass filter banks circumventing the correspondence problem that arises from EMD. The decision which type of signal decomposition should be applied depends mainly on the objective of further analyses of signal components. In the case of nonlinear questions, the advantages of EMD-based approaches are obvious due to the preservation of nonlinear signal properties (in contrast to linear filtering procedures). The decision in favor of fixed or adaptive signal decompositions in any other case of further processing depends on the inter-individual variance of frequency components, which has to be evaluated subjectively. Furthermore, when group analyses are targeted, individual frequency variations make the application of narrowband filters difficult. EMD-based approaches enable a fully automated and individual decomposition of frequency components and reduce effects of subjective definitions of frequency ranges of interest. On the other hand, the flexibility of EMD-based approaches is achieved at the expense of the need for solving the correspondence problem. Thus in summary, a set of fixed bandpass filters is less intricate if prior knowledge about frequency components is available, stable frequency components predominate in group data, and nonlinear signal properties are not of interest. If any of these prerequisites is not fulfilled EMD-based signal decompositions provide a serious option knowing well that those approaches entail the correspondence problem.

From the methodological perspective the proposed approach for the solution of the CP for multiple sets of IMFs is based on the transformation of the entire assignment problem into a set of classic pairwise assignment problems. Thus the specification of a reference set, to which all other IMF sets are assigned, seems to be inevitable. Certainly in general, the final assignment depends on the choice of the reference set, which may present a drawback of the proposed methodology. However based on our experience, the effects are minor when an IMF set of maximum cardinality is used as a reference. In addition, discrepancies between different reference sets are mainly related to assignments of IMFs carrying noise proportions or very low frequency signal parts with low amplitudes, which could be considered as residuals. Finally the assignment of substantially relevant IMFs is very robust with respect to refer-

![Figure 9](#) Results of further analysis of the assigned IMFs derived from EEG data. (A) Average amplitude spectra of groups C (mainly IMFs 3) – F (mainly IMFs 6) of the reference-based multiple assignment are given. (B) Averaged envelopes of groups C– F are depicted (mean standard deviation).
ence set modifications. The main advantage in comparison to clustering-based approaches is that due to the matching conditions on bipartite graphs, the stringent separation of different IMF sets is a priori guaranteed. Additionally to hierarchical clustering we also used density-based spatial clustering of applications with noise (DBSCAN [34]) and k-means clustering. In the case of the simulated data set both methods underperform in comparison to the hierarchical clustering. Moreover, it turned out that the clustering result of DBSCAN is very sensitive to the choice of the distance threshold used for the definition of nearby neighbors. The obvious advantage that DBSCAN does not require an a priori specification of the number of clusters is accompanied with the need of a very cautious adjustment of in the context of solving the CP. In addition, the appropriate selection of is strongly data dependent, which makes a fully automatic matching of IMFs more difficult. These findings were substantiated for the real data set. Indeed, both k-means and DBSCAN perform worse and yield partial assignments, which are not in agreement with plausible manual assignments. Yet, DBSCAN allows the assignment of IMFs to a noise class, if there are not enough similar IMFs of other signals to form clusters. This property may offer potential for solving CPs of type (iv).

The current study focuses on the conditioning of the CP types (i) and (ii) (Figure 3), which can be e.g. tackled with an extension of the KMA. Problems of type (iii) are in particular interesting because a matching of a single IMF to several IMFs of another set could provide evidence for the occurrence of mode mixing. The determination of such 1:n assignments is known as the generalized assignment problem, which is a combinatorial optimization problem with high complexity (generally NP-hard). Since the number of IMFs per signal decomposition is thought to be low, presumably exact methods might be applied. Generally, the assignment problem may be formulated as an integer linear programming problem [16, 36]. Problems of type (iv) are known as k-cardinality assignment problems. With the correspondence problem type D, IMFs without compatibility to the other sets of IMFs may be neglected. Promising approaches may include threshold concepts, where high cost assignments (in our case matches of minor spectrally correlated IMFs) are not considered [37].

According to [10], we investigated alternative metrics and cost functions in addition to those presented. Briefly, approaches using l-norms are inferior in comparison to correlation-based methodologies, which arise from the impact of amplitude differences between the spectra. In contrast, correlation-based distances or costs are independent of amplitudes and target only spectral distributions, respectively. Moreover, we integrated the cosine distance instead of one minus Pearson’s correlation coefficient for both the reference and the clustering-based methods. This ultimately resulted in very similar results. For a significantly stronger penalization of low correlations we also alternatively embedded the cost function \((1 - \max(r, 0))/\max(r, 0)\) for the reference-based approach. In summary, the differences between using \((1 - \max(r, 0))/\max(r, 0)\) or \(1 - r\) were only marginal because spectral correlations of matching IMF pairs were much larger than zero in the majority of cases. However, for data sets yielding smaller correlations, a nonlinear transformation of correlations into costs (including infinite costs) seems to be appropriate.

The continued application of the proposed methodological developments and their steady advancements in the field of biomedical signal analysis will lead to improved automated processing concepts. The EMD can also be used to derive information about natural frequency components and the frequency ranges in which they occur. In this way filters with fixed cut-off frequencies can be designed which filter characteristics corresponding to the frequency content of the IMFs and, additionally, which allow an artifact-free instantaneous frequency and phase computation. This is essential for a reliable CFCA of the EEG. The pattern-related analysis of the neonatal EEG aims at an improved understanding of those processes which may be related to brain maturation. In the clinical setting HRV analysis might contribute to the detection of root causes of sudden unexpected death in epilepsy and could help determine reliable parameters for seizure detection and prediction.

6. Conclusions

We have presented a novel approach to solve the correspondence problem associated with EMD analyses. The algorithm may be universally applied for various EMD types as well as for multivariate, multi-trial, or multiple subject analyses, respectively. By construction, it is possible to integrate different types of pairwise assignment problems, which enables a flexible adaptation to particular conditions in practical applications.

Acknowledgments

This study was supported by the German Research Foundation (Wi 1166/12-1/2 and Le 2025/6/1-2).

References


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