Keywords
Bayes’ rule, Caraguel-Vanderstichel nomogram, ruler, diagnostic test

Summary
Objective: To present a geometric ruler to determine predictive values of binary diagnostic test results from prevalence, sensitivity and specificity without calculation.

Methods: On logarithmic scale the relationships between pre-/post-test odds and the likelihood ratio as well as sensitivity, specificity and the likelihood ratio appear as simple linear (additive) equations.

Results: Each of these additive equations can be solved geometrically in form of a ruler with three scales. By amalgamation we devise a novel non-electronic tool which omits the intermediate step of likelihood ratio determination.

Conclusions: We propose a simple geometric method to aid in interpretation of diagnostic test results for both practical and educational purposes.

1. Introduction
The predictive value of a diagnostic test result can be calculated using Bayes’ Rule. Fagan proposed a nomogram which offers the possibility to determine the result geometrically [1]. As an alternative, Hellmich and Lehmacher proposed a ruler for interpreting diagnostic test results [2]. Both tools, the nomogram and the ruler, assume knowledge of the likelihood ratio (LR) of the test result. To overcome this limitation in case of a binary test, Caraguel and Vanderstichel proposed a two-step nomogram which integrates the calculation of the LR from given sensitivity and specificity [3].

We propose a novel ruler which skips the likelihood ratio calculation. After abridging the arithmetic on which the ruler is based, two versions of the tool will be presented, exemplified and discussed.

2. Methods
We are interested in patients who may or may not have a condition of interest (C, either C+ or C−), e.g. a disease, and try to classify them by means of a binary diagnostic test T (either T+ or T−). A diagnostic test usually comes with information about its sensitivity, i.e. the probability that the test result will be positive on a patient having the condition of interest [Sens = P(T+ | C+)] and the specificity, i.e. the probability that the test result will be negative on a patient not having the condition [Spec = P(T− | C−)]. The positive predictive value (PV+), which can be interpreted as the post-test probability [P(C+ | T+)] for having the condition, can be calculated from sensitivity, specificity and prevalence [P(C+)] by means of Bayes’ rule:

\[ PV+ = \frac{P(T+ | C+) \times P(C+)}{P(T+ | C+) \times P(C+) + P(T+ | C−) \times P(C−)} \]

The negative predictive value \[ PV− = \frac{P(C− | T−)}{P(T− | C−) \times P(C−) + P(T− | C+) \times P(C+)} \]

With some algebraic transformation of equation (E1) we obtain a useful relationship between the likelihood ratio for a positive test result \[ LR+ = \frac{P(T+ | C+)}{P(T+ | C−)} = \frac{(1 – Sens)}{Spec} \]

\[ LR+ \times \text{pre-test odds} \]

Thus, the post-test odds equal the likelihood ratio times the pre-test odds. The same equation holds for a negative test result \[ LR− = \frac{P(T− | C−)}{P(T− | C+) \times P(T− | C−) + P(T− | C−) \times P(C+)} \]

The multiplicative equation (Eq. 3) becomes easily solvable geometrically, if we take the logarithm of both sides:
Based on this equation (Eq. 4), Fagan designed a nomogram to geometrically determine the post-test odds from a given LR and pre-test odds. Similarly, Hellmich and Lehmacher proposed a ruler for interpreting diagnostic tests.

In the frequent case, that the LR of a test is not reported, but only the sensitivity and specificity are given, a preliminary step is needed to determine its value. From the general definition of the likelihood ratio \( LR = P(T|C+)/P(T|C–) \) we obtain by taking logarithms:

\[
\log(LR) = \log(Sens) - \log(1−Spec) \quad (5a)
\]

\[
\log(LR) = \log(1 – Sens) – \log(Spec) \quad (5b)
\]

These linear (additive) equations (Eqs. 5a, b) may, again, easily be solved geometrically.

Caraguel and Vanderstichel designed a two-step nomogram for the ad-hoc interpretation of a diagnostic test result. It completes the Fagan's nomogram by a first step, in which the LR is determined for given sensitivity and specificity. In the second step, the resulting LR is used to find the post-test probability for given prevalence. In contrast to the Fagan's nomogram, whose construction is easy, the two-step version is definitely of higher complexity. It consists of five axes, of which two contain two scales. A straight edge is necessary to connect three scales per step to determine the results. In the following section we present the basics of a ruler which extends the device proposed by Hellmich and Lehmacher.

3. Results

The extended ruler, which offers the possibility to determine the likelihood ratio and post-test probability given sensitivity, specificity and prevalence, is presented in Figure 1. The extended ruler consists of three axes. The first one (A) is made of two inversely arranged logarithmic scales corresponding to the sensitivity and specificity of the test. This axis has labels on both sides. If the LR+ is to be calculated, the sensitivity can be read off from the right and the specificity from the left side. Else if the LR- is to be calculated, the two sides are reversed. The second axis is made of a logarithmic scale and displays the likelihood ratio. The third axis (B) consists of a logarithmic odds scale which displays pre- and post-test probabilities. The tool is a direct extension from the ruler presented by Hellmich and Lehmacher which is included in the right part of the figure. The extended

Figure 1 Pattern for cut and paste of an extended ruler to calculate the post-test probability via the likelihood ratio from sensitivity, specificity and pre-test probability (prevalence)
ruler works this way: The likelihood ratio scale in the middle is fixed and the other two scales on its right and left are adjusted laterally. ▶ Figure 1 gives a pattern for cut and paste where axis A and B can be cut out and placed inside the dotted lines at the respective side.

To illustrate use of the extended ruler we consider the same example as Caraguel and Vanderstichel, the MRI screening test for breast cancer in high-risk female patients [3, 5]. The test has a sensitivity of 75% and a specificity of 96%. We will now consider a patient coming from a high risk population with a prevalence of 2%. First, we will look at the computation of the likelihood ratio for a positive test result (LR+) and the associated post-test probability. The ruler works in nearly the same way for LR–.

The first step is to align, by lateral adjustment, the likelihood ratio of 1 with a specificity of 96% on axis A and a pre-test probability of 2% on axis B. For LR+ the specificity is displayed on the left side of axis A. The next step is to draw a horizontal line through the sensitivity, here 75%, given on the right scale of axis A. Alternatively, any straight edge can be used for this means. The third and last step is to read off the results. The likelihood ratio is provided by the middle axis, here approximately 19. The value is greater than 10 and, thus, the observed test result has a high diagnostic value. Next, the post-test probability to have cancer can be found on axis B with a value of approximately 28%. If the likelihood ratio for a negative test result needs to be determined, the ruler works exactly the same way, except that the legend of axis A is now reversed. Here a likelihood ratio (LR–) of approximately 0.26 is associated with a post-test probability for cancer of 0.53% (▶ Figure 2).

Not in every case the likelihood ratio for a diagnostic test is reported. Moreover, not every practitioner is familiar with the meaning or use of these likelihood ratios. However, all tools discussed so far assume such knowledge. We will demonstrate that the extended ruler has one more significant advantage over Fagan’s nomogram. The extended ruler is able to obtain the post-test result directly from sensitivity and specificity for a given prevalence, eliminating the intermediate step of the likelihood ratio determination and use.

In ▶ Figure 3, a “smart” variation of the extended ruler is presented where the likelihood ratio scale was dropped with only two axes remaining. Axis B on the right hand site is fixed, presenting pre- and post-test probabilities, axis A on the left hand

![Figure 2](https://example.com/image2.png)  
**Figure 2** Worked example to illustrate the use of the extended ruler based on MRI screening test for breast cancer, compare Caraguel and Vanderstichel (2013)

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site is flexible and corresponds to the sensitivity and specificity of the test, with differing labels, depending on whether computation for a positive or negative test result needs to be performed.

Figure 4 shows how the post-test probability for a negative test result can be obtained with the extended “smart” ruler. The steps for obtaining the result are similar to the ones before. Firstly, the specificity needs to be aligned with the pre-test probability. Secondly, a horizontal line needs to be drawn through the sensitivity. On its intersection with axis B, the post-test probability can be read off. For a negative test result, the sensitivity can be found on the left and the specificity on the right scale of axis A.

4. Discussion

The use of the original Fagan's nomogram was limited, since the initial step for computation of the likelihood ratio was missing. The two-step nomogram offers a solution for this, but at the price of higher complexity. Instead of five axes the “smart” extended ruler requires only three. Moreover, the result is easy to understand, since the change in (un-)certainty is given on the same probability scale. Last but not least, the handling is simpler, although personal preferences may vary on this point.

Methods for diagnostic testing are continuously being refined. For example, Beinish and, recently, Hughes elaborate on the use of “information graphs” as a means of characterizing diagnostic test performance [6, 7]. The proposed ruler supports the understanding of the relationship between pre- and post-test probability. Ughi and Jassó emphasize the generally higher level of involvement with non-virtual approaches and their power as ‘imperfect’ tools [8], thus the use of a geometric device may be preferred to a computer program (or smart phone application), particularly for educational purposes.

Though both rulers may also be designed as a circular disk, the linear version is presented here as it offers the possibility for immediate use following cut and paste. We hope our tool proves useful in clinical and educational practice.

Instructions for use:

Decide if computation for a positive or negative test result (T+/T-) needs to be done. For T+ the top labels of axis A are valid, for T- the bottom ones.

1. Align the Specificity (A) with the Pre-Test Probability (B)
2. Draw a horizontal line from the Sensitivity (A) to the Post-Test Probability Scale (B)
3. Read off the Post-Test Probability (B)

Figure 3 Pattern for cut and paste of the “smart” extended ruler where the likelihood ratio scale was dropped


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5. Conclusion

The simple geometrical approach provided may support the interpretation of diagnostic test results, both for practical and didactic purposes. A next step should be an evaluation study on this question. Moreover, while nomograms to calculate sample size in diagnostic testing have been proposed [9], interval estimation seems to be a promising area of construction.

References


Figure 4 Worked example to illustrate the use of the “smart” extended ruler