Dear Editor,

Consider a diagnostic test resulting in a binary classification of subjects. The proportions correctly classified among subjects actually positive (sensitivity) and subjects actually negative (specificity) are properties of the test in question. The quantity sensitivity + specificity – 1 is Youden’s index for rating diagnostic tests [1]. By 1950, when Youden’s index was first described [2], the use of Bayes’ theorem by Turing and colleagues had proved crucial in cryptographically work during World War II [3, 4] (incidentally initiating a renaissance for statistical applications of Bayesian methods), and Shannon’s groundbreaking work providing the basis for information theory had recently been published [5]. In order to describe how these analyses are linked, a little notation and terminology is first required. To describe a test, two groups of subjects (cases (D+) and controls (D−)) are identified by means of a gold standard, independent of the test in question. Then, all subjects in both groups are tested and the outcomes (positive (T+) or negative (T−)) recorded (Youden’s analysis in its original form uses a binary classification of subjects and of test outcomes). Although diagnostic tests are imperfect, a large proportion of subjects who are cases should provide a positive test outcome (these results are true positives); while a small proportion of controls will also provide a positive test outcome (these are false positives). The proportion of cases testing positive (sensitivity, true positive proportion, denoted TPP) is an estimate of the conditional probability Pr(T+|D+), and the proportion of controls testing positive (1−specificity, false positive proportion, denoted FPP) is an estimate of Pr(T+|D−).

Then Youden’s index is \( J = \text{TPP} − \text{FPP} = \Pr(T+|D+) − \Pr(T+|D−) \).

Now consider Shannon’s analysis, and in particular the information content of a message (such as the outcome of a test). Briefly, if a positive test outcome were considered to be very unlikely before the result of the test became known, the information content of a positive outcome – when realized – would be large. And conversely, if a positive test outcome were considered likely, the realized information content of that outcome would be small. Shannon’s analysis defines the information content of a message \( h(p) \) as a function of the probability of that message \( p \): \( h(p) = −\log(p) \) (the base of the logarithm can be chosen to suit the application). Then \( h(p) \) declines monotonically from \( +\infty \) to 0 as \( p \) increases from 0 to 1, so meeting the requirements as stated. In Figure 1, Shannon’s information graph is shown, with \( \Pr(T+|D+) = \text{TPP} \) and \( \Pr(T+|D−) = \text{FPP} \) marked on the probability axis (\( \text{TPP} > \text{FPP} \) is assumed, but otherwise the numerical values are not important here). The corresponding values on the information axis are \( −\log(\Pr(T+|D+)) \) and \( −\log(\Pr(T+|D−)) \), respectively. On the horizontal (probability) axis, the interval \( \text{TPP} − \text{FPP} = \Pr(T+|D+) − \Pr(T+|D−) = J \). The corresponding interval on the vertical (information) axis is \( −\log(\Pr(T+|D−)) − (−\log(\Pr(T+|D+))) \) = \( \log(\Pr(T+|D+)/\Pr(T+|D−)) \). This quantity is the log-likelihood ratio of a positive test outcome, denoted here \( \log[LR(+)] \). Now note that Bayes’ theorem may be stated as follows: the log-likelihood ratio is added to the initial log-odds to obtain the final log-odds. Turing and colleagues, who referred to the log-likelihood ratio as the weight of evidence, adopted this format; as did Van den Ende and colleagues in the

**Keywords**

Youden’s index, log-likelihood ratio, Bayes’ theorem, diagnostic test

**Summary**

By means of Shannon’s relationship between information and probability, Youden’s index for rating diagnostic tests is shown to be a probability-scale analogue of the log-likelihood ratio of a positive test outcome.

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context of diagnostic clinical epidemiology [6, 7].

The correspondence between $J$ and $\log[LR(+)]$ is such that there is a monotonic increasing trend in $\log[LR(+)]$ from $J = 0$ ($\log[LR(+)] = 0$) towards $J = 1$. This brings within the Bayesian paradigm the observation that $\Pr(D+|T+)$ (the posterior probability that a subject is a case given a positive test outcome) > $\Pr(D+)$ (the prior probability that a subject is a case) if and only if $J > 0$ (a “process qualification” requirement for a test used for diagnosis) [8]. Between the extremes, there is not a one-to-one correspondence between $J$ and $\log[LR(+)]$ because diagnostic tests with equal values of $J$ will not necessarily have equal values of $\log[LR(+)]$. Notwithstanding, the correspondence is likely to be of interest, for example in studies concerning the evaluation of diagnostic tests where both Youden’s index and likelihood ratios are calculated [9, 10]. Previously, Zhou et al. noted informally that Youden’s index “reflects the likelihood of a positive result among patients with versus without the condition” [1]. The present analysis now provides an analytical underpinning for that description. In words: Youden’s index is a probability-scale analogue of the log-likelihood ratio of a positive test outcome.

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References